國立金門技術學院 99 學年度第 2 學期四技日間部轉學考試**答案卷**

系別／年級：營建工程系二年級

科　　目：微積分

評分欄

1. $\lim_{x\to 7}\frac{\sqrt[5]{3-5x}}{(x-5)^3}=\frac{\lim_{x\to 7}\sqrt[5]{3-5x}}{\lim_{x\to 7}(x-5)^3}=\frac{\sqrt[5]{\lim_{x\to 7}(3-5x)}}{\lim_{x\to 7}(x-5)^3}=\frac{\sqrt[5]{3-35}}{(7-5)^3}=\frac{-2}{8}=-\frac{1}{4}$

2. $\lim_{x\to 3^+}f(x)=\lim_{x\to 3^+}\frac{x-[|x|]}{x-1}=\lim_{x\to 3^+}\frac{x-3}{x-1}=0$

$\lim_{x\to 3^-}f(x)=\lim_{x\to 3^-}\frac{x-[|x|]}{x-1}=\lim_{x\to 3^-}\frac{x-2}{x-1}=\frac{1}{2}$

因 $\lim_{x\to 3^-}f(x)\neq\lim_{x\to 3^+}f(x)$, 故 $\lim_{x\to 3}f(x)$ 不存在

3.
$$
\begin{aligned}
f'(x)&=\frac{d}{dx}[(5x+6)(4x^3-3x+2)]\\
&=(5x+6)\frac{d}{dx}(4x^3-3x+2)+(4x^3-3x+2)\frac{d}{dx}(5x+6)\\
&=(5x+6)(12x^2-3)+5(4x^3-3x+2)\\
&=80x^3+72x^2-30x-8
\end{aligned}
$$

4.令 $f(x)=x^5$ 且 $g(x)=2x^2+3x+1$

$f'(x)=5x^4, g'(x)=4x+3,$

則 $\frac{d}{dx}[(2x^2+3x+1)^5]=\frac{d}{dx}[f(g(x))]=f'(g(x))g'(x)$

$=5[(2x^2+3x+1)^4(4x+3)$

5.
$$
\begin{aligned}
\frac{dy}{dx}&=\frac{dy}{du}\times\frac{du}{dx}=\frac{d}{du}(u^2+1)\times\frac{d}{dx}(\frac{1}{x^2})\\
&=(3u^2)(-\frac{2}{x^3})=3(\frac{1}{x^2})^2(-\frac{2}{x^3})\\
&=-\frac{6}{x^7}
\end{aligned}
$$

6. $f'(x)=\sqrt{x}+(x-2)\dfrac{1}{2\sqrt{x}}=\dfrac{3x-2}{2\sqrt{x}}$

在$(0,4)$中，f 的臨界數為$\dfrac{2}{3}$

因 $f(0)=0$，$f(\dfrac{2}{3})=-\dfrac{4\sqrt{6}}{9}$，$f(4)=4$ 故 $f(4)>f(0)>f(\dfrac{2}{3})$

所以 極大值為 4，極小值為$-\dfrac{4\sqrt{6}}{9}$

7. $f'(x)=\dfrac{d}{dx}(x^{4/3}-4x^{1/3})=\dfrac{d}{dx}(x^{4/3})-4\dfrac{d}{dx}(x^{1/3})=\dfrac{4}{3}x^{1/3}-\dfrac{4}{3}x^{-2/3}$

$f''(x)=\dfrac{d}{dx}(\dfrac{4}{3}x^{1/3}-\dfrac{4}{3}x^{-2/3})=\dfrac{4}{9}x^{-2/3}+\dfrac{8}{9}x^{-5/3}=\dfrac{4}{9}x^{-5/3}(x+2)$

$x<-2$	-2	$-2<x<0$	0	$x>0$
$f''(x)>0$，上凹	$f''(-2)=0$	$f''(x)<0$，下凹	$f''(x)=0$，不存在	$f''(x)>0$，上凹

故 f 圖形在$(-\infty,-2)$與$(0,\infty)$為上凹，在$(-2,0)$為下凹，

而反曲點分別為$(-2,6\sqrt[3]{2})$與$(0,0)$

8. $\int\dfrac{x-1}{\sqrt{x}+1}dx=\int\dfrac{(x-1)(\sqrt{x}-1)}{(\sqrt{x}+1)(\sqrt{x}-1)}dx=\int\dfrac{(x-1)(\sqrt{x}-1)}{x-1}dx$

$=\int(\sqrt{x}-1)dx=\int\sqrt{x}dx-\int dx=\dfrac{2}{3}x^{3/2}-x+C$

9. $\dfrac{d}{dx}\int_x^2\sqrt{t+1}dt=\dfrac{d}{dx}(-\int_2^x\sqrt{t+1}dt)=-\dfrac{d}{dt}\int_2^x\sqrt{t+1}dt=-\sqrt{x+1}$

10. 証明：令$u=1-x$，則 $du=-dx$ ；當$x=0$時$u=1$，當$x=1$時$u=0$

$\int_0^1 f(1-x)dx=\int_1^0 -f(u)du=\int_0^1 f(u)du=\int_0^1 f(x)dx$

故 $\int_0^1 f(x)dx=\int_0^1 f(1-x)dx$

1