

國立金門技術學院 99 學年度第 1 學期四技日間部轉學考試參考答案

系別／年級：營建工程系二年級

科 目：微積分

1. 求極限 $\lim_{x \rightarrow 0} \frac{\sqrt{x+1}-1}{x}$

Sol :

$$\lim_{x \rightarrow 0} \frac{\sqrt{x+1}-1}{x} = \left(\frac{\sqrt{x+1}-1}{x} \right) \left(\frac{\sqrt{x+1}+1}{\sqrt{x+1}+1} \right) = \frac{(x+1)-1}{x(\sqrt{x+1}+1)} = \frac{x}{x(\sqrt{x+1}+1)} = \frac{1}{\sqrt{x+1}+1}, x \neq 0$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{x+1}-1}{x} = \lim_{x \rightarrow \infty} \frac{1}{\sqrt{x+1}+1} = \frac{1}{1+1} = \frac{1}{2}$$

2. 求 $y = 2x \cos x - 2 \sin x$ 的導函數

$$\begin{aligned} \text{Sol : } \frac{dy}{dx} &= (2x) \left(\frac{d}{dx} [\cos x] \right) + (\cos x) \left(\frac{d}{dx} [2x] \right) - 2 \frac{d}{dx} [\sin x] \\ &= (2x)(-\sin x) + (\cos x)(2) - 2(\cos x) = -2x \sin x \end{aligned}$$

3. 求 $f(x) = x^4$ 的圖形在 $x = -1$ 的斜率

$$\text{Sol : } f(x) = x^4 \text{ 導函數為 } f'(x) = 4x^3, \text{ 當 } x = -1 \text{ 時, 所求的斜率為 } f'(-1) = -4$$

4. 求 $y = \left(\frac{3x-1}{x^3+3} \right)^2$ 的導函數

$$\begin{aligned} \text{Sol : } y' &= 2 \left(\frac{3x-1}{x^3+3} \right) \frac{d}{dx} \left[\frac{3x-1}{x^3+3} \right] = \left[\frac{2(3x-1)}{x^3+3} \right] \left[\frac{(x^3+3)(3) - (3x-1)(2x)}{(x^3+3)^2} \right] \\ &= \frac{2(3x-1)(3x^2+9-6x^2+2x)}{(x^3+3)^3} = \frac{2(3x-1)(-3x^2+2x+9)}{(x^3+3)^3} \end{aligned}$$

5. 求函數 $f(x) = 2x - 3x^{\frac{2}{3}}$ 在閉區間 $[-1, 3]$ 上的極值

Sol: 將 $f(x)$ 微分得到 $f'(x) = 2 - \frac{2}{x^{1/3}} = \left(\frac{x^{1/3} - 1}{x^{1/3}} \right)$

函數之臨界數為 0, 1, 極值之判斷如表

左端點	臨界數	臨界數	右端點
$f(-1) = -5$ 極小值	$f(0) = 0$ 極大值	$f(1) = -1$	$f(3) \approx -0.245$

6. 求 $\int \frac{1}{4x-1} dx$

Sol: 令 $u = 4x - 1$, 則 $du = 4dx$

$$\int \frac{1}{4x-1} dx = \frac{1}{4} \int \left(\frac{1}{4x-1} \right) 4dx = \frac{1}{4} \int \frac{1}{u} du = \frac{1}{4} \ln |u| + C = \frac{1}{4} \ln |4x-1| + C$$

7. 求 $\int \frac{2x}{(x+1)^2} dx$

Sol: 令 $u = x + 1$, 則 $du = dx$ 和 $x = u - 1$ 代入

$$\begin{aligned} \int \frac{2x}{(x+1)^2} dx &= \int \frac{2(u-1)}{u^2} du = 2 \int \left(\frac{u}{u^2} - \frac{1}{u^2} \right) du = 2 \int \frac{du}{u} - 2 \int u^{-2} du = 2 \ln |u| - 2 \left(\frac{u^{-1}}{-1} \right) + C \\ &= 2 \ln |u| + \frac{2}{u} + C = 2 \ln |x-1| + \frac{2}{x+1} + C \end{aligned}$$

8. 解微分方程式 $\frac{dy}{dx} = \frac{1}{x \ln x}$

Sol: $y = \int \frac{1}{x \ln x} dx = \int \frac{1/x}{\ln x} dx = \int \frac{u'}{u} dx = \ln |u| + C = \ln |\ln x| + C$

9. 解 $50e^{-x} = 30$

Sol :

$$50e^{-x} = 30$$

$$e^{-x} = \frac{30}{50}$$

$$\ln e^{-x} = \ln \frac{3}{5}$$

$$-x = \ln \frac{3}{5}$$

$$x = -\ln \frac{3}{5}$$

$$x = \ln \frac{5}{3}$$

$$x = \ln 5 - \ln 3 \cong 0.511$$

11. 求 $y = \ln(1 + e^{2x})$ 的導函數。

Sol : 令 $u = 1 + e^{2x}$; $u' = 2e^{2x}$

$$y' = \frac{1}{1 + e^{2x}} (2e^{2x}) = \frac{2e^{2x}}{1 + e^{2x}}$$