

科目：數學

$$1. p = \frac{13 \times \binom{4}{3} \times 12 \times \binom{4}{2}}{\binom{52}{5}}$$

$$2. p = \frac{0.6 \times 0.3}{0.4 \times 0.5 + 0.6 \times 0.3}$$

$$3. E(X) = 1 \times 1/6 + 2 \times 1/6 + 3 \times 1/6 + 4 \times 1/6 + 5 \times 1/6 + 6 \times 1/6 = 21/6 = 7/2$$
$$E(X^2) = 1^2 \times 1/6 + 2^2 \times 1/6 + 3^2 \times 1/6 + 4^2 \times 1/6 + 5^2 \times 1/6 + 6^2 \times 1/6 = 91/6$$
$$\text{Var}(X) = E(X^2) - E(X)^2 = 91/6 - 49/4$$

$$4. E(X) = 5$$
$$\text{Var}(X) = (7-3)^2 / 12$$

$$5. f_X(x) = \frac{10}{3} x(1-x^3), 0 \leq x \leq 1$$
$$f_Y(y) = 5y^4, 0 \leq y \leq 1$$

$$6. A^{-1} = \begin{bmatrix} 4 & -3 & 1 \\ -2 & 1 & 0 \\ 5 & -4 & 1 \end{bmatrix}.$$

$$7. x_1 = 1, x_2 = 0, x_3 = 1$$

$$8. v_1 = (1, 2, 2), v_2 = (0, 2, -2), v_3 = \left(\frac{14}{9}, \frac{-7}{18}, \frac{-7}{18}\right).$$

$$9. \lambda_1 = 1, X_1 = \begin{bmatrix} 2 \\ 6 \\ -1 \end{bmatrix}, \lambda_2 = 2, X_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \lambda_3 = 9, X_3 = \begin{bmatrix} 14 \\ 2 \\ 7 \end{bmatrix}.$$

$$10. \sin A = \frac{1}{2} \begin{bmatrix} \sin 2 + \sin 8 & -\sin 2 + \sin 8 \\ -\sin 2 + \sin 8 & \sin 2 + \sin 8 \end{bmatrix}.$$