

國立金門技術學院

98 學年度第 2 學期四技日間部轉學考試

營建工程系三年級參考答案

微積分

1. Sol :

$$\lim_{x \rightarrow 0} \frac{\sqrt{x+1}-1}{x} = \left(\frac{\sqrt{x+1}-1}{x} \right) \left(\frac{\sqrt{x+1}+1}{\sqrt{x+1}+1} \right) = \frac{(x+1)-1}{x(\sqrt{x+1}+1)} = \frac{x}{x(\sqrt{x+1}+1)} = \frac{1}{\sqrt{x+1}+1}, x \neq 0$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{x+1}-1}{x} = \lim_{x \rightarrow \infty} \frac{1}{\sqrt{x+1}+1} = \frac{1}{1+1} = \frac{1}{2}$$

2. Sol : $\frac{dy}{dx} = (2x) \left(\frac{d}{dx} [\cos x] \right) + (\cos x) \left(\frac{d}{dx} [2x] \right) - 2 \frac{d}{dx} [\sin x]$
 $= (2x)(-\sin x) + (\cos x)(2) - 2(\cos x) = -2x \sin x$

3. Sol : $f(x) = x^4$ 導函數為 $f'(x) = 4x^3$, 當 $x = -1$ 時, 所求的斜率為 $f'(-1) = -4$

4. Sol : $y' = 2 \left(\frac{3x-1}{x^3+3} \right) \frac{d}{dx} \left[\frac{3x-1}{x^2+3} \right] = \left[\frac{2(3x-1)}{x^2+3} \right] \left[\frac{(x^2+3)(3) - (3x-1)(2x)}{(x^2+3)^2} \right]$
 $= \frac{2(3x-1)(3x^2+9-6x^2+2x)}{(x^2+3)^3} = \frac{2(3x-1)(-3x^2+2x+9)}{(x^2+3)^3}$

5. Sol : 將 $f(x)$ 微分得到 $f'(x) = 2 - \frac{2}{x^{1/3}} = \left(\frac{x^{1/3}-1}{x^{1/3}} \right)$

函數之臨界數為 0,1 , 極值之判斷如表

左端點	臨界數	臨界數	右端點
$f(-1) = -5$	$f(0) = 0$	$f(1) = -1$	$f(3) \approx -0.245$
極小值	極大值		

6. Sol: 一階導函數: $f'(x) = \frac{20x}{(x^2-4)^2}$

二階導函數: $f''(x) = \frac{-20(3x^2+4)}{(x^2-4)^3}$

x 截距: $(-3,0), (3,0)$

y 截距: $(0, 9/2)$

鉛直漸進線: $x = -2, x = 2$

水平漸進線: $y = 2$

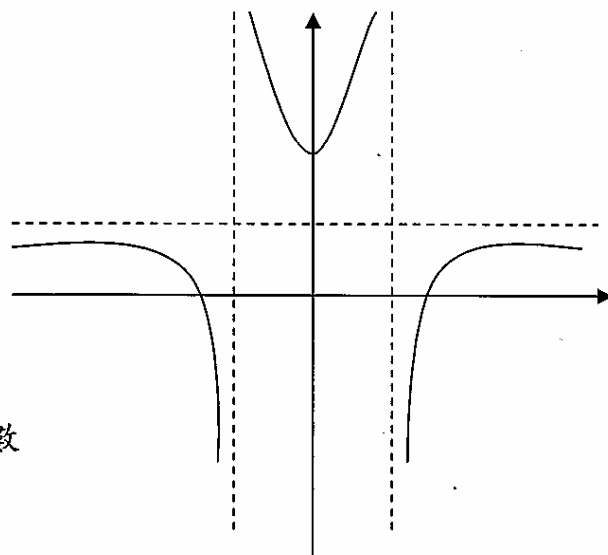
臨界數: $x = 0$

可能的反曲點: 無

定義域: 除了 $x = \pm 2$ 的所有實數

對稱: 對稱於 x 軸

檢測區間: $(-\infty, -2), (-2, 0), (0, 2), (2, \infty)$



7. Sol: $\frac{d}{dx}[(\ln x)^3] = 3(\ln x)^2 \frac{d}{dx}[\ln x] = 3(\ln x)^2 \frac{1}{x}$

8. Sol: 令 $u = 4x - 1$, 則 $du = 4dx$

$$\int \frac{1}{4x-1} dx = \frac{1}{4} \int \left(\frac{1}{4x-1} \right) 4dx = \frac{1}{4} \int \frac{1}{u} du = \frac{1}{4} \ln |u| + C = \frac{1}{4} \ln |4x-1| + C$$

9. Sol: 令 $u = x+1$, 則 $du = dx$ 和 $x = u-1$ 代入

$$\int \frac{2x}{(x+1)^2} dx = \int \frac{2(u-1)}{u^2} du = 2 \int \left(\frac{u}{u^2} - \frac{1}{u^2} \right) du = 2 \int \frac{du}{u} - 2 \int u^{-2} du = 2 \ln |u| - 2 \left(\frac{u^{-1}}{-1} \right) + C$$

$$= 2 \ln |u| + \frac{2}{u} + C = 2 \ln |x-1| + \frac{2}{x+1} + C$$

10. Sol : $y = \int \frac{1}{x \ln x} dx = \int \frac{1/x}{\ln x} dx = \int \frac{u'}{u} dx = \ln |u| + C = \ln |\ln x| + C$



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