

# 國立金門技術學院

98 學年度第 2 學期四技日間部轉學考試

營建工程系二年級參考答案

## 微積分

1. Sol: 先求  $f'(x) = 2x + 2$ ，則斜率為  $f'(-5) = -8$

$$\text{又 } f(-5) = (-5)^2 + 2(-5) - 6 = 9$$

故切線方程式為  $(y - 9) = -8(x + 5)$

2. Sol:

$$\lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{x} = \left( \frac{\sqrt{x+1} - 1}{x} \right) \left( \frac{\sqrt{x+1} + 1}{\sqrt{x+1} + 1} \right) = \frac{(x+1) - 1}{x(\sqrt{x+1} + 1)} = \frac{x}{x(\sqrt{x+1} + 1)} = \frac{1}{\sqrt{x+1} + 1}, x \neq 0$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{x} = \lim_{x \rightarrow \infty} \frac{1}{\sqrt{x+1} + 1} = \frac{1}{1+1} = \frac{1}{2}$$

- 3.

$$\begin{aligned} \text{Sol: } \frac{dy}{dx} &= (2x) \left( \frac{d}{dx} [\cos x] \right) + (\cos x) \left( \frac{d}{dx} [2x] \right) - 2 \frac{d}{dx} [\sin x] \\ &= (2x)(-\sin x) + (\cos x)(2) - 2(\cos x) = -2x \sin x \end{aligned}$$

- 4.

Sol:  $f(x) = x^4$  導函數為  $f'(x) = 4x^3$ ，當  $x = -1$  時，所求的斜率為  $f'(-1) = -4$

- 5.

$$\text{Sol: } y' = \frac{1 - \cos x}{\sin x} = \frac{(\sin x)(\sin x) - (1 - \cos x)(\cos x)}{\sin^2 x} = \frac{\sin^2 x + \cos^2 x - \cos x}{\sin^2 x} = \frac{1 - \cos x}{\sin^2 x}$$

$$\text{或 } y' = -\csc x \cot x + \csc^2 x$$

6.

$$\begin{aligned} \text{Sol: } y' &= 2 \left( \frac{3x-1}{x^3+3} \right) \frac{d}{dx} \left[ \frac{3x-1}{x^2+3} \right] = \left[ \frac{2(3x-1)}{x^2+3} \right] \left[ \frac{(x^2+3)(3) - (3x-1)(2x)}{(x^2+3)^2} \right] \\ &= \frac{2(3x-1)(3x^2+9-6x^2+2x)}{(x^2+3)^3} = \frac{2(3x-1)(-3x^2+2x+9)}{(x^2+3)^3} \end{aligned}$$

7.

$$\text{Sol: 將 } f(x) \text{ 微分得到 } f'(x) = 2 - \frac{2}{x^{1/3}} = \left( \frac{x^{1/3} - 1}{x^{1/3}} \right)$$

函數之臨界數為 0,1，極值之判斷如表

左端點	臨界數	臨界數	右端點
$f(-1) = -5$ 極小值	$f(0) = 0$ 極大值	$f(1) = -1$	$f(3) \approx -0.245$

8.

$$\text{Sol: } \lim_{x \rightarrow \infty} \frac{2x-1}{x+1} = \lim_{x \rightarrow \infty} \frac{2x-1}{\frac{x}{x+1}} = \lim_{x \rightarrow \infty} \frac{2 - \frac{1}{x}}{1 + \frac{1}{x}} = \frac{\lim_{x \rightarrow \infty} 2 - \lim_{x \rightarrow \infty} \frac{1}{x}}{\lim_{x \rightarrow \infty} 1 + \lim_{x \rightarrow \infty} \frac{1}{x}} = \frac{2-0}{1+0} = 2$$

9.

$$\text{Sol: 一階導函數: } f'(x) = \frac{20x}{(x^2-4)^2}$$

$$\text{二階導函數: } f''(x) = \frac{-20(3x^2+4)}{(x^2-4)^3}$$

$x$  截距:  $(-3,0), (3,0)$

$y$  截距:  $(0, 9/2)$

鉛直漸進線:  $x = -2, x = 2$

水平漸進線:  $y = 2$

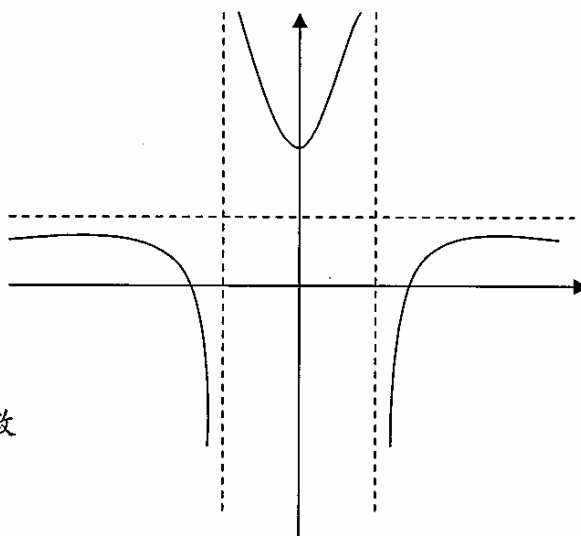
臨界數:  $x = 0$

可能的反曲點: 無

定義域: 除了  $x = \pm 2$  的所有實數

對稱: 對稱於  $x$  軸

檢測區間:  $(-\infty, -2), (-2, 0), (0, 2), (2, \infty)$



10. Sol : (0,2)至  $y=4-x^2$  圖上一點  $(x,y)$  的距離為  $d=\sqrt{(x-0)^2+(y-2)^2}$

$$\text{故 } d = \sqrt{(x-0)^2+(y-2)^2} = \sqrt{x^2+(4-x^2-2)^2} = \sqrt{x^4-3x^2+4}$$

$$\text{令 } f'(x)=0, \text{ 得 } f'(x)=4x^3-6x=2x(2x^2-3)=0$$

$$x=0, \sqrt{3/2}, -\sqrt{3/2}$$

故距離最近之點為  $(\sqrt{3/2}, 5/2), (-\sqrt{3/2}, 5/2)$



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